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TITLE: Inhomogeneous Diophantine approximation of some Hurwitzian numbers

ABSTRACT: In this talk we report on joint work with Richard Bumby.

The real number θ is called Hurwitzian if after finitely many terms its sequence of partial quotients has the form

$$f_1(0), \ldots, f_R(0), f_1(1), \ldots, f_R(1), \ldots \ ,$$

where each $f_j(x)$ is a polynomial with rational coefficients that takes positive integer values on the positive integers. For example, quadratic irrationals are Hurwitzian numbers (of order 0) and $e^{2/k}$ is a Hurwitzian number (of order 1) for every nonzero integer k.

The inhomogeneous approximation constant for a pair of real numbers θ, ϕ (where $\phi \notin \mathbb{Z}\theta + \mathbb{Z}$) is defined to be

$$M(\theta, \phi) = \liminf_{|q| \to \infty} \Big\{ |q| \, \|q\,\theta - \phi\| \; : \; q \in \mathbb{Z} \Big\},$$

where $\|x\|$ denotes the distance from the real number x to the nearest integer. In the middle of the twentieth century there was substantial work related to these inhomogeneous approximation constants as well as to the associated inhomogeneous Markoff values.

In the last decade, interest in these problems has re-kindled. In particular, Takao Komatsu used several different types of continued fractions to compute inhomogeneous constants of the form $M(e^{1/s}, \phi)$ for $\phi \in \mathbb{Q}\, e^{1/s} + \mathbb{Q}$. In this talk we make use of the "relative rationality" of these pairs θ, ϕ to show how the technically simpler ideas of Grace [*Proc. London Math. Soc.* **17** (1918), p. 316-319] and regular simple continued fractions can be used to unify and extend Komatsu's results. Among the new results for Hurwitzian θ is a characterization of $\phi \in \mathbb{Q}\,\theta + \mathbb{Q}$ for which $M(\theta, \phi) = 0$, as well as a proof of the conjecture that $n^2\, M(e^{1/s}, 1/n)$ equals 0 or 1/2 for all positive integers n, s.